# CONDENSATION OF VAPOR ON A COLD FLUID JET

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UDC 532.522:621.175

The heat transfer that accompanies vapor condensation on the surface plane of a supercooled fluid jet expelled at a high velocity into a vapor-filled space is studied analytically.

The concept of expelling a cold fluid jet into a gas-filled space is used in a number of technological processes as well as in jet apparatus (injectors) and heat exchangers. The current interest in the study of physical phenomena occurring at the jet surface is explained by the high intensity of heat-, mass-, and momentum-transfer from the vapor to the fluid. Of particular interest for practical technological design is the determination of the coefficient of heat transfer of the vapor with a moving fluid, with allowance for the phase transition at the surface. A similar problem was first studied in [1] in a boundary layer approximation for the case of a "motionless" vapor-filled space, within the framework of the "old" Prandtl mixing path theory. Later, the condensation of moving vapor on a cold fluid jet was studied in [2]. Exact determination of the turbulent structure coefficient of the jet is currently not possible, owing to a lack of experimental data and the complexity of the physical processes in the turbulent mixing layer accompanied by phase transition at the surface. The use of an indicated empirical constant obtained from tests with homogeneous incompressible mixing layers leads to results nearly twice the experimental values [3]. This discrepancy was to be expected, since even a difference in the density of the mixing flows alone leads to pronounced changes of every single mixing-layer parameter [4]. The influence of the turbulent-structure parameter on the heat-transfer intensity in the condensation of vapor on a cold fluid jet is studied in the present paper.

The following problem (Fig. 1) is examined. A cold fluid with given and constant thermophysical parameters occupies the lower half-space and moves at a constant velocity $u_{flow}$. Owing to condensation, a turbulent mixing layer develops at the boundary with the vapor-filled space. It is assumed that complete instantaneous condensation of the vapor occurs at the upper boundary of the mixing layer. This makes it possible to examine the resulting flow in an incompressible-fluid approximation, without considering the density of the heat sources in the boundary layer. The concept of a "mixing-layer boundary" is purely conditional, since the actual boundary between the fluid and the vapor varies randomly in time. Exact determination of the boundary conditions requires knowledge of the correlations between the transfer parameters and the instantaneous position of the elementary portion of the boundary in space. Since there is a complete lack of experimental data on this question, it is assumed that the "effective" boundary of condensation is a certain plane at which the corresponding laws of mass, momentum, and energy flow density are fulfilled. The lower boundary of the mixing layer is defined similarly.

It is further assumed that the vapor has given and constant thermophysical parameters and that it moves normal to the condensation surface at a velocity $v_{vap}$.

This formulation of the problem corresponds to the condensation of vapor on a fluid jet expelled from a nozzle, whereas the formulation of the problem in [1] corresponds more to a cold jet expelled from a hole in a smooth wall.

The abscissa axis is taken along the condensation boundary. Usually, the direction of the longitudinal boundary-layer axis coincides with that of the main-flow velocity. However, for small condensation angles ($\beta \approx 1$ to $5°$), the question of selecting the system of coordinates is not essential from the viewpoint of an exact description of the flow, since boundary-layer theory itself is an approximate model of the actual flow. In the system of coordinates selected, the boundary conditions can be expressed in a much simpler way.

All-Union Scientific-Research Structural Design Institute of the Petroleum-Refining and Petroleum-Chemical Industry, Kiev. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 15, No. 6, pp. 994-999, December, 1968. Original article submitted January 29, 1968.

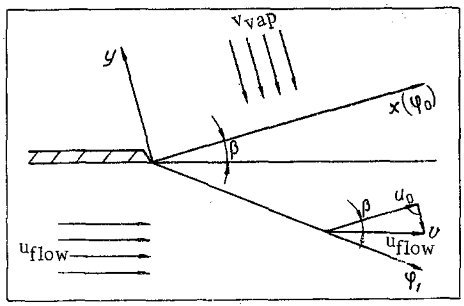

Fig. 1

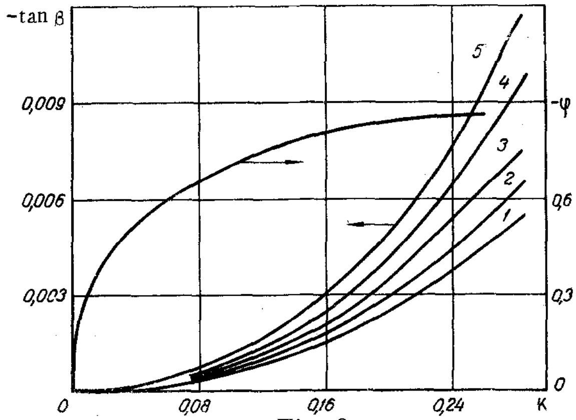

Fig. 2

Fig. 1. Schematic drawing of jet boundary layer.

Fig. 2. Plot of $\tan\beta$ and $\varphi$ vs the heat parameter K: 1) $a$ = 0.08; 2) 0.10; 3) 0.12; 4) 0.14; 5) 0.16.

Further, it is necessary to take into consideration the fundamental assumptions of the "old" Prandtl mixing-path theory [1], and to remember that the condensing vapor decelerates the upper layers of the moving fluid ($\partial u/\partial y < 0$). As a result, one arrives readily at a Tolmien-type equation

$$F''' - F = 0. \tag{1}$$

The boundary-layer velocity components can be expressed in terms of the function F:

$$\begin{aligned} u &= u_0 F'(\varphi), \\ v &= a u_0 [\varphi F'(\varphi) - F(\varphi)], \end{aligned} \tag{2}$$

where $u_0 = u_{flow} \cos\beta$; $a$ is the turbulent-structure coefficient of the mixing layer; and $\varphi = y/ax$ is a dimensionless coordinate of the boundary layer.

The solution of equation (1) has the form

$$F = C_1 e^{\varphi} + C_2 e^{-\frac{\varphi}{2}} \cos \frac{\sqrt{3}}{2}\varphi + C_3 e^{-\frac{\varphi}{2}} \sin \frac{\sqrt{3}}{2}\varphi. \tag{3}$$

To solve the problem, it is necessary to determine the three constants of integration $C_1$, $C_2$, $C_3$, the position of the condensation boundary (angle $\beta$), the boundary layer thickness ($\varphi_1$), the vapor velocity $v_{vap}$, and the pressure difference between the gas-filled space and the fluid.

In the following, subscript 1 refers to values of function F and its derivatives at the lower boundary of the mixing layer, and subscript 0 to its values at the condensation boundary. The density of the medium at the inner boundary $\varphi = \varphi_1$ is continuous, so that the velocity is also continuous:

$$u = u_0 \Longrightarrow F_1' = 1, \tag{4}$$

$$v = -u_0 \tan\beta \Longrightarrow \tan\beta = -a[\varphi_1 - F_1]. \tag{5}$$

The relation (5) is derived on the basis of (4). When relations (4) and (5) are satisfied, it follows from the condition for the continuity of the momentum flow density across the boundary $\varphi_1$ that the turbulent shearing stresses are zero for $\varphi = \varphi_1$

$$\tau(\varphi_1) = 0 \Longrightarrow \left.\frac{\partial u}{\partial y}\right|_{y \in \varphi_1} = 0 \Longrightarrow F_1'' = 0. \tag{6}$$

At the upper limit of the boundary layer ($\varphi = 0$), the condition for the continuity of the mass flow density across the condensation surface must be satisfied

$$\{\rho u_k n_k\} = 0.$$

Here, and in the following, brackets denote the difference between values on the right and left side of the condensation boundary, $n_k$ are the components of the unit vector of the outer normal to the phase transition boundary, and $u_k$ are the velocity components. After simple calculations, it is easy to obtain

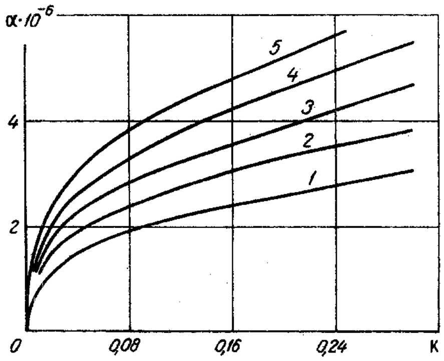

Fig. 3

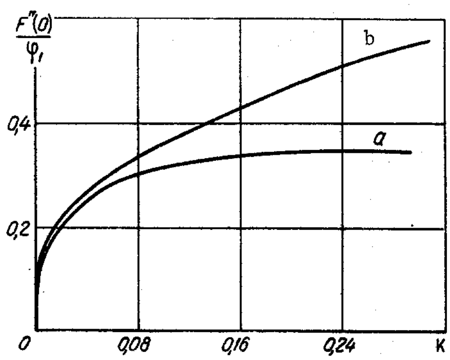

Fig. 4

Fig. 3. Plot of the heat-transfer coefficient $\alpha$ vs K and $a$ (for u = 20 m/sec and $c_p \rho = 4.187 \cdot 10^6$ J/m$^3$·deg = 1000 kcal/m$^3$·deg). For 1-5, see Fig. 2.

Fig. 4. Plot of $F''(0)/\varphi_1$ vs the heat parameter K: a) data of [1]; b) author's results.

$$\gamma v_{vap} = -a u_0 F_0, \tag{7}$$

where $\gamma = \rho_{vap}/\rho_{fluid}$.

The continuity equation for the momentum flow density (the influence of viscous stresses is neglected)

$$\{\Pi^T_{ik} n_k\} = 0,$$

where,

$$\Pi^T_{ik} = \rho u_i u_k + P\delta_{ik} + \rho\overline{u_i u_k};$$
$$\delta_{ik} = 1 \text{ for } i = k; \quad \delta_{ik} = 0 \text{ for } i \neq k,$$

leads to two equations:

$$uv + \overline{uv} = 0,$$

$$P_{fluid} - P_{vap} = \rho_{vap} v^2_{vap} - \rho_{fluid} v^2_{fluid} - \rho_{fluid} \overline{v^2_{fluid}}.$$

Here, it was assumed that the vapor velocity is normal to the condensation surface and that the vapor-filled space is free of turbulent pulsations. Making use of Prandtl's hypothesis about the existence of a relationship between Reynolds stresses and the mean characteristics of the flow [1], and bearing in mind that $\partial u/\partial y < 0$ in the case under consideration, the first of the equations obtained can be reduced to the form

$$-F_0' F_0 = \frac{1}{2} F_0''^2. \tag{8}$$

The second equation can be used for determining the pressure jump at the phase transition boundary.

It remains to examine the continuity equation for the energy flow density. The following conditions are assumed to be satisfied: the molecular heat conductivity of the fluid is negligible compared to the convective conductivity; dissipative processes which lead to additional heating of the fluid play an insignificant role; and the kinematic components in the expression for the total heat capacity are small compared to the thermal components ($u^2/2 + i \approx i$). Under these assumptions, we have the following equation

$$\{(\rho u_k i + \rho\overline{u_k i}) n_k\} = 0,$$

where i is the specific enthalpy of the vapor or fluid.

Allowing for the continuity of the mass-flow density, we easily obtain

$$v_{fluid}(i_{vap} - i_{fluid}) = c_p \overline{v' T'}.$$

With the aid of simple assumptions and transformations similar to those performed in [1], the equation obtained can be reduced to the form

$$K F_0'' = -\varphi_i F_0, \tag{9}$$

TABLE 1. Parameters of the Turbulent Layer at the Boundaries as a Function of Its Thickness $\varphi_1$

| $-\varphi_1$ | $-\tan\beta$ | $-F(\varphi_1)$ | $-F(0)$ | $-F''(0)$ | $\frac{-F''(0)}{\varphi_1}$ | $K$ | $\alpha\cdot 10^{-4}$ | $v_n$ |
|---|---|---|---|---|---|---|---|---|
| 0,1 | 0,0000058 | 0,100073 | 0,000086 | 0,0050060 | 0,05006 | 0,00171 | 28,8 | 0,23 |
| 0,2 | 0,0000067 | 0,200083 | 0,000284 | 0,020024 | 0,1001 | 0,002834 | 57,5 | 0,761 |
| 0,3 | 0,0000087 | 0,300109 | 0,001123 | 0,045108 | 0,15036 | 0,00747 | 86,3 | 3,00 |
| 0,4 | 0,0000170 | 0,400212 | 0,003418 | 0,080419 | 0,20105 | 0,01700 | 115,5 | 9,14 |
| 0,5 | 0,0000505 | 0,500631 | 0,008468 | 0,12635 | 0,2597 | 0,03351 | 145 | 22,7 |
| 0,6 | 0,000166 | 0,602082 | 0,018594 | 0,183844 | 0,3064 | 0,06003 | 176 | 49,6 |
| 0,7 | 0,000530 | 0,706619 | 0,037114 | 0,255298 | 0,3647 | 0,10176 | 209 | 99,4 |
| 0,8 | 0,001663 | 0,820787 | 0,074024 | 0,347920 | 0,4349 | 0,17021 | 250 | 198 |
| 0,9 | 0,006569 | 0,982109 | 0,174737 | 0,500389 | 0,5583 | 0,31397 | 320 | 466 |

Note: $u = 20$ m/sec; $c_p \rho = 1000$ kcal/m$^3\cdot$deg; $a = 0.08$.

TABLE 2. Dimensionless Longitudinal Velocity F"($\varphi$) and Relative Turbulent Shearing Stress $F''^2(\varphi)$ as a Function of the Instantaneous Coordinate $\varphi$ for $\varphi_1 = -0.9$

| $-\varphi$ | $F'(\varphi)$ | $F''^2(\varphi)$ | $-\varphi$ | $F'(\varphi)$ | $F''^2(\varphi)$ |
|---|---|---|---|---|---|
| 0,1 | 0,7665 | 0,185 | 0,7 | 0,984 | 0,0313 |
| 0,3 | 0,8532 | 0,172 | 0,9 | 1 | 0 |
| 0,5 | 0,930 | 0,0966 | | | |

where

$$K = \frac{c_p (T_0 - T_1)}{i_{vap} - i_0}.$$

The solution of the problem formulated is physically acceptable for the following constraints:

1) $\varphi_1 < 0$, according to the formulation of the problem;

2) $\partial u/\partial y < 0 \Rightarrow F''(\varphi) < 0$ over the entire boundary layer;

3) $u > 0 \Rightarrow F''(\varphi) > 0$.

It is natural to seek the solution of the problem for a given value of the heat parameter K. However, owing to the nature of the equations obtained, it is more convenient to take the position of the lower limit $\varphi_1$ of the mixing layer as the initial parameter. The constants of integration are determined from the system of equations (4), (6), (8), the position of the condensation boundary from Eq. (5), and the vapor velocity from Eq. (7). From Eq. (9), one can obtain the corresponding value of K.

The heat-transfer coefficient for vapor condensation on a cold fluid jet can be calculated from [1]:

$$\alpha = 3600\, a u_0 c_p \rho \frac{F''(0)}{\varphi_1}. \tag{10}$$

The results of the computation are compiled in Table 1. The values of the dimensionless longitudinal velocity $F'(\varphi)$ and the relative values of the shearing and turbulent stresses $F''^2(\varphi)$ for $K = 0.314$ and $\varphi_1 = -0.9$ are shown in Table 2.

The condensation angle $\beta$ is plotted vs the parameters K and $a$ in Fig. 2.

From Fig. 3, it is easy to observe the combined influence of the parameters K and $a$ on the heat-transfer coefficient $\alpha$.

Naturally, the difference in the formulation of the problems has resulted (Fig. 4) in a discrepancy between our data and the results obtained in [1].

In conclusion, it should be noted that condensation of vapor on a cold fluid jet can occur not only at the surface of the mixing layer but also in its central portions. It would be thus of interest to study heat-transfer processes in mixing layers with allowance for "bulk" condensation.

## NOTATION

$u$ is the longitudinal velocity component;
$u_{flow}$ is the flow velocity;
$v$ is the transverse velocity component;
$\rho_{vap}$ is the vapor density;
$\rho_{fluid}$ is the fluid density;
$P_{vap}$ is the vapor pressure;
$P_{fluid}$ is the pressure in the fluid;
$\varphi$ is the dimensionless coordinate of the mixing layer;
$\beta$ is the condensation angle;
$\Pi_{ik}^{T}$ is the tensor of momentum flow density;
$K$ is the heat parameter of the problem;
$a$ is the turbulent-structure parameter of the jet;
$\alpha$ is the coefficient of heat transfer between vapor and fluid;
$c_p$ is the heat capacity of the fluid;
$T_0$ is the fluid temperature at the condensation boundary;
$T_1$ is the temperature of the main fluid flow;
$i_{vap}$ is the specific enthalpy of the vapor;
$i_{fluid}$ is the enthalpy of the fluid at the condensation boundary.

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